Whose saving behavior really matters in the long run?
The Pasinetti (irrelevance) theorem revisited

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Abstract

This review paper is intended to outline some of the main qualitative theoretical issues involved in the debates on the results emerging from the Pasinetti (irrelevance) theorem, which is an important element of the post-Keynesian approach to growth and distribution. Firstly, it is briefly described the Cambridge (U.K.) *vs* Cambridge (U.K.) controversy following the publication of the original works by Kaldor and Pasinetti. It is then reviewed the subsequent Cambridge (U.S.) *vs* Cambridge (U.K.) exchange between Samuelson and Modigliani, on one side, and Pasinetti, Robinson, and Kaldor on the other side.

Palavras-chave
comportamento de poupança, crescimento, distribuição.

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Resumo

Este artigo-resenha descreve algumas das principais questões teóricas envolvidas nos intensos debates que se seguiram aos resultados oriundos do teorema (da irrelevância) de Pasinetti, que é um elemento importante da abordagem pós-keynesiana do crescimento e da distribuição. Inicialmente, descreve-se brevemente a controvérsia Cambridge (RU) vs *Cambridge (RU) que se seguiu à publicação dos trabalhos originais de Kaldor e Pasinetti. Em seguida, resenha-se o debate Cambridge (EUA)* vs *Cambridge (RU) subseqüente, que opôs, de um lado, Samuelson e Modigliani, e, de outro, Pasinetti, Robsinson e Kaldor.*

1_ Introduction

This review paper is intended to outline some of the main qualitative theoretical issues involved in the debates on the results emerging from the so-called Pasinetti (irrelevance) theorem, which is a major result in the post-Keynesian approach to growth and distribution. To put it more precisely, it is intended to focus on two different stages of that debate. Firstly, the Cambridge (U.K.) *vs* Cambridge (U.K.) controversy following the publication of Kaldor (1955-1956) and Pasinetti (1962), which involved Meade (1963 and 1966) and Meade and Hahn (1965) against Pasinetti (1964 and 1966a). Secondly, the Cambridge (U.S.) *vs* Cambridge (U.K.) exchange between Samuelson and Modigliani (1966a and 1966b), on one side, and Pasinetti (1966b), Robinson (1966), and Kaldor (1966), on the other side.

To put it briefly, the essence of the Pasinetti theorem (1962) is the following. In a model economy where there exists a group who receives only property income (capitalists), the wealth of that group will grow at a constant rate, depending on their saving rate and their rate of return earned. In this case, long-run equilibrium is defined as a situation where the distribution of wealth remains constant; the wealth of all other groups, and so of the economy as a whole, must also grow at this same rate. If, in addition, it is assumed that the economy grows at a constant (natural) rate, it follows that the profit rate of as well as the profits share will depend only on the saving preferences of capitalists, while technology and saving preferences of all other groups (workers) are irrelevant.[1]

2_ First act: Cambridge (U.K.) vs Cambridge (U.K.)

Meade's (1963) main purpose is to discuss two propositions concerning the rate of profit in a steady-state growth which appear in the neo-Keynesian approach, namely:

1. that the rate of growth of the output depends upon the rate of population growth and the rate of technical progress, and is otherwise independent of the form of the production function;
2. that the profit rate depends solely upon the rate of output growth and upon the thriftiness conditions in the economy and is otherwise independent of the form of the production function.

1 Since this paper is intended to focus primarily upon the earlier qualitative theoretical issues involved in these debates, and due to (binding) space constraints, the reader is directed to the original works for a complete development of the mathematical counterpart of those issues. I would recall that most of these original works and the immediately subsequent related literature, whic is not discussed in the compass of this paper, were gathered in the volume edited by Carlo Panico and Neri Salvadori (1993).

As Meade and Hahn (1965) were to recall later on, Meade (1963) was concerned to demonstrate the neoclassical result that the very long run rate of profit in a golden age will depend upon technology if in that age all people both work and own property.

In order to consider those neo-Keynesian propositions in the neoclassical system, Meade (1963) assumes a simple Cobb-Douglas production function. After several algebraic manipulations, he shows that with unitary elasticities of substitution between the factors of production added to Pasinetti's (1962) assumptions, the neoclassical result will be true if $s_V \leq (s_W / 1 - Q)$, where s_V stands for the capitalists' (who do not work and save a fixed proportion of their income) propensity to save, S_W for the workers' propensity to save and Q for the marginal product of labor. On the other hand, the neo-Keynesian result will be true if $s_V > s_W /(1 - Q)$. As for the two neo-Keynesian propositions stated above, Meade concludes that they are dependent

1. upon two special assumptions about the production function, namely, that there are only two factors of production and that it is the Harrodian and not the Hicksian rate of technical progress which is independent of the other features of the production function;
2. upon the assumption that the thriftiness conditions are such that the ratio of total savings to income, s, can be expressed as a constant proportion of the marginal product of capital, u.

In his reply to Meade (1963), Pasinetti (1964) emphasizes that one must take a Cobb-Douglas production function as necessarily representing the technical characteristics of the real world in order to accept Meade's conclusions, a view that Pasinetti is not inclined to take. For Pasinetti, Meade's constant procedure consists in singling out a sort of critical value of some parameter and then in saying: if the parameter is less than so much, the neo-Keynesian result holds and if , on the other hand, the parameter is more than so much, the neoclassical result holds. Pasinetti argues that such procedure gives the reader an impression of symmetry which does not correspond to the substance of Meade's results, for the latter's two alternatives are profoundly asymmetrical. More precisely, when Meade says "the neo-Keynesian result holds" the actual meaning is "the neo-Keynesian result holds in general", that is, both within the neo-Keynesian and within the neoclassical theoretical framework. On the other hand, when he says that "the neoclassical result holds" the actual meaning, it is replied, the actual meaning "only within the limits of neoclassical assumptions".

Pasinetti claims that this point must be explicitly pointed out because when Meade arrives at results which differ from his, Meade's conclusions depend crucially on:

1. the rejection of Pasinetti's (1962) restriction that the workers' saving propensity must be smaller than the ratio of investment to income ($s_W < I/Y$), meaning that workers' total saving are less than required (equilibrium) investment;
2. the assumption of infinite substitutability between labor and capital so that, no matter how small workers' saving propensity might be, workers' savings alone will suffice to keep up full employment investment.

Therefore, Pasinetti replies, Meade's statement about the (aforementioned) conditions under which whether neo-Keynesian or neoclassical results will hold is incorrect; for Meade has not added to, but rather replaced, the original assumptions. Actually, in Pasinetti's view, to introduce the restriction that the workers' saving propensity must be smaller than the ratio of investment to income in fact implies that the neoclassical result is never true.

To phrase it another way, let us recall that a distinctive property of the Pasinetti theorem is that, within the range in which it is valid, that is, $s_W < I/Y$, it is independent of any assumption about whether and how technology (in the sense of the capital-output ratio) is influenced by changes in the profit rate. Indeed, Meade (1963) accepted this result, although he preferred to state that restriction in terms of savings, instead of in terms of investments, by writing $s_W > s_c\,(P/Y)$, where P/Y is the ratio of profit to income (in Meade's terminology $1-Q$). Meade then proceeded to examine the case in which that restriction is reversed and claimed that in such case the long-run equilibrium growth which would eventually follow is such that the capital-output ratio (and not the rate of profits) is equal to the natural rate of growth divided by the workers' saving propensity.

In this context, Pasinetti's (1964) contention is that this second, neoclassical result – the neo-Keynesian one – does not have general validity. For, as soon as we investigate the range in which the inequality $s_W > s_c\,(P/Y)$ is reversed, assumptions about technology become relevant, and the conclusions depend crucially on the assumptions. That is, when this inequality is satisfied, the relation defining the equilibrium profit rate follows, whatever one's assumptions about production technology might be. However, when the same inequality is reversed, as Meade does, his results do not necessarily follow, unless it is added the assumption of a well-behaved production function allowing substitution between capital and labor in response to changes in the profit rate. It would be possible, say, to make other (quite reasonable) assumptions on technology, under which Meade's result would never follow. In our view, it is essentially this very asymmetry that Pasinetti's reply to Meade was meant to point out.

In their rejoinder to Pasinetti's (1964) reply, Meade and Hahn (1965) recall that Meade (1963) was concerned to show that the long-run profit rate in a golden age will depend upon technology if, in that age, all people both work and own property. Moreover, he also gave a simple account of how such a stage might come about even if, at some initial date, there were people who only owned wealth and did not work. Hence, Meade and Hahn interpret Pasinetti's (1964) reply as

i. claiming that these results depend on the assumption of infinite substitutability between labor and capital;
ii. stressing that he had in any case assumed that people who both work and save can never save enough to allow full employment growth.

Their purpose is then to argue that the claim under (i) above is false and that, in an important respect, the assumption under (ii) above begs the real question at issue.

In order to (presumably) get as far away as possible from any implication of infinite substitutability between capital and labor, they consider a world of many goods where a finite number of discrete production processes are available. Following Pasinetti (1962), they suppose that there are two groups of people, pure capitalists and those who both work and own wealth. If they are accumulating property at different proportionate rates, the asset distribution will vary so that three possibilities exist for a long-run equilibrium situation where further changes in assets distribution cease to play a role:

a. the proportion of assets owned by workers may become negligibly small;
b. the asset distribution may remain constant through time;
c. the proportion of assets owned by pure capitalists may become negligibly small.

In cases (a) and (b), it is argued, the eventual rate of profit multiplied by the capitalist's saving propensity will equal the rate of growth; this is Pasinetti's case. But in case (c), savings will eventually become proportional to income, and this is the case investigated by Meade (1963).

To examine the analytical implications of such dependence, let us recall that the golden age natural growth rate, g_n, must be equal to the growth rate of capital stock. If savings (= investment) is a fixed proportion, s_p, of profits, P, then $s_p\, P/K = I/K = g_n$, and hence $P/K = g_n/s_p$, where K is the capital stock. When, however, I is a fixed proportion, s_y, of income, Y, we have $s_y Y/K = I/K$, so that $K/Y = g_n$. In other words, the income-capital ratio, but not the profit-capital ratio, is now independent of technology. In this case, they maintain, to know the equilibrium profit rate, we must therefore solve for the whole system, which then will involve the technological aspects of the economy. Moreover, they claim that case (c) does not depend upon the assumption of infinite substitutability between labor and capital, but is compatible with the (book-of-blueprints) technology assumed by them.

They argue that Pasinetti actually ruled out case (c) by his assumption that the saving propensity of those who both work and own capital, s_W, is less than the ratio of full-employment investment to income (I/Y), an assumption, they believe, begs the question. For in golden-age growth, we have $I/K = g_n$ and also $I/Y = g_n(K/Y)$, which means that the condition assumed by Pasinetti, $s_W < I/Y$, can be alternatively expressed as the assumption that in the golden age we have $s_W < g_n(K/Y)$ or $(K/Y) > (s_W/g_n)$. This assumption, however, could legitimately be made a priori – without investigating the general conditions of the golden-age equilibrium, which will depend on demand conditions and relative prices as well as upon technological possibilities – only in case the book of blueprints contained no set of activities which is sufficiently labor intensive to reduce the capital-output ratio below the given level g_n/s_W.

This might be so if s_W was negligibly small or if the available technology did not include any very labor-intensive methods. If this were the case, they maintain, Pasinetti's results would apply. More precisely, if we happened to start from a position in which a very small proportion of property were owned by the pure capitalists, then in these conditions the rate of investment would not be sufficient to keep the growth of capital stock in line with the growth of labor, even when the most intensive-intensive techniques were chosen, and unemployment would result. For full employment, the real wage rate would have to be reduced until the distribution of income were shifted to profits until the savings propensity of the community were sufficient to correspond to a rate of investment which generated a sufficient supply of real capital to employ all the available labor. To put it another way, the equilibrium value of the marginal product of labor Q would not be great enough to make $s_p(1-Q) < s_W$. But they raise the following question: what if in the real world the minimum technologically possible value of K/Y is less than s_W/g_n? In their view, Pasinetti could, of course, legitimately build a model in which this is just assumed not to be the case. But it may be the case, and what Meade (1963) did was simply to claim the right to examine the possibility.

As mentioned above, Pasinetti's (1964) reply to Meade (1963) was meant to point out a clear asymmetry between his results and Meade's. For Pasinetti (1966a), even though Meade and Hahn (1965) had restated those problems in terms of a book-of-blueprints approach instead of a smooth production function, their results are subject to exactly the same asymmetry. Actually, it is pointed out, the necessity of postulating a well-behaved production function allowing substitution between capital and labor in response to changes in the profit rate is not discussed by them, their purpose simply seeming to be that of denying the necessity of postulating infinite substitutability.

For Pasinetti, even to make this mild claim, a further restriction is necessary, and one that does destroy the alleged symmetry between the range in which the neo-Keynesian condition $s_W < s_c(P/Y)$ is satisfied and the range in which such condition is reversed (the neoclassical case). He argues that this restriction is implicitly introduced, as seen above, when they consider the case $s_W > g_n k^*$ – in their terminology $s_W < g_n(K/Y)$ – where k^* is the capital-output ratio entailed by the least capital-intensive technique which is known. In Pasinetti's view, one should realize that when the restriction $s_W > g_n k^*$ is imposed, one can no longer simply talk of reversal of the inequality given by $s_W < s_c(P/Y)$. More precisely, one must talk of reversal of the latter supplemented by the former. Notice that reversal of the latter and nonfulfillment of the former provides, by the way, an example in which the latter is reversed, and yet Meade's results do not follow. Pasinetti also maintains that no such restriction appears in Meade's (1963) original analysis. There, the results are presented as valid (for any positive rate of growth) whatever value, however small, s_W may have, except that it should be nonnegative. It is with reference to this claim – Pasinetti (1966a) readily emphasizes – that the term infinite was used in Pasinetti (1964), for in order to make it one has to assume infinite substitutability between capital and labor.

3_ Second act: Cambridge (U.S.) vs Cambridge (U.K.)

The purpose of the study and elucidation of the Pasinetti's theorem set forth by Samuelson and Modigliani (1966a, hereafter SM), are threefold . First, it is intended to show the limited range of the workers and capitalists saving coefficients within which that theorem is valid. Outside that range, SM believe to have formulated a theorem that is dual to it – and of the same generality. It too involves a paradox, namely, that the average product of the capital – the reciprocal of the capital-output ratio – to which the system will settle is, this time, equal to g_n/s_W and completely independent of the propensity to save out of profits of the pure capitalists or the form of production function. On the other hand, all the other golden-age variables of the system depend only upon g_n/s_W and on the form of production functions so that the complete duality of all these results with the Pasinetti theorem would be notable.

Second, it is intended to dispel the (presumed by SM) erroneous notion that Pasinetti's analysis has some peculiar relevance to a Kaldorian alternative theory of distribution or to some version of a Cambridge theory of distribution. SM point out that, as Pasinetti himself made clear, his analysis is one of the greatest generality, in the sense that his theorem applies in fact to any system capable of a golden-age growth path. For SM, it is precisely because of this great generality that Pasinetti's analysis can, in no way, help us to define income distribution. To make this point clear, their analysis deal primarily with a neoclassical production function capable of a smooth factor substitution and with the case of perfectly competitive markets, under which conditions competition will enforce, at all times, equality of factor prices to factor marginal productivities.

Thirdly, SM investigate – and claim to have proved – the stability of the Pasinetti golden age in the case where it is valid. To put it another way, SM intend to prove that the system will asymptotically approach the steady state from arbitrary initial conditions, at least in a local neighborhood of that state. And where their anti-Pasinetti golden age is argued to hold, in which the workers' saving propensity is all important, they intend to demonstrate its global asymptotic stability.

After some algebraic manipulations of their production function-type system, SM come up with an equation representing (what would be) the basic result of the Pasinetti theorem, $f'(k^*) = r^* = g_n/s_c$, where k^* and r^* stand for the steady state equilibrium values of the capital-intensive ratio and the rate of profit, respectively. In turn, the nonnegativity conditions given by $k_W \geq 0$ and $k_c \geq 0$, can be easily shown to imply $s_W < s_c$ and $s_W \leq a\,(k^*)sc = g_n[k^*/f(k^*)]$, respectively, where k_W stands for the workers' capital, k_c for the capitalists' capital and $a(k) = rk/f(k)$, for the income share accruing to capital. It is claimed that the latter inequality is more stringent than the former, since the capital share, $a(k)$, is generally less than one, and empirically very much less. Thus, if $a(k^*) = 0.25$ and $S_c = 0.2$, Pasinetti's theorem, when derived in this way, could not hold for sS_W higher than a modest 0.05. Besides, SM claim that the latter inequality has some correspondence to Pasinetti's restriction given by $S_W < I/K$, to the extent that in the steady state given by $I/K = (dK/dt)(1/K) = (dL/dt)(1/L) = g_n$, we have $I/K = g_n K/Y = g_n [k/f(k)]$.

In their view, however, Pasinetti's simple inequality is ill-defined, for outside of the steady state, I/Y could take any value whatever. Furthermore, even on the equilibrium growth path, the expression $f(k)/k$ is not a given of the problem but a characteristic of the solution, if any. Except possibly in the very special case of fixed production coefficients, where K/Y might be identified with the technologically determined (minimum) capital coefficient. For SM, their inequality $s_W \leq a(k^*)s_c = g_n [k^*/f(k^*)]$, in turn, has the merit of making explicit what must not be left ill-defined: that the inequality given by $s_W \leq a(k^*)s_c$ must hold precisely at $k = k^*$, the k that corresponds to $r^* = g_n/s_c$.

The main conclusion, drawn by SM so far, is therefore that the numerical range of the parameter S_W, for which Pasinetti's theorem is applicable, is severely limited. Besides, they argue that the behavior of the system out of this range is covered by a theorem complementary to Pasinetti's theorem, its Dual. In order to prove their Dual theorem – which states that the average product of capital or the reciprocal of the capital-output ratio, to which the system will settle, is equal to g_n/s_W and independent of the propensity to save out of profits or the form of production function – SM start by examining what happens when $k_c \geq 0$ fails to hold, when $s_W > g_n k^*/f(k^*) = g_n/A(k^*) = a(k^*)s_c$, where $A(k^*) = K/Y$ stands for the average product of capital. In this case, eventually the rate of growth of capitalists' assets will become and remain smaller than the rate of growth of workers' assets and also smaller than g_n. This in turn means that, asymptotically, Kc/K – the capitalists' share of total wealth – will approach zero while the workers' share, K_W/K, will approach unity; with k_c tending to vanish, the limiting behavior of the system then reduces to the familiar

Solow-Swan process with a single class of savers, namely the workers. Once workers' saving starts dominating, we are then in the domain of the anti-Pasinetti dual theorem. In such a situation, profit rate, capital-labor, and capital-output ratios and, therefore, also the distribution of income between wages and profits are fully independent of the capitalists' propensity to save. But while the average product of capital (and its reciprocal the capital-output ratio) is independent, even the form of production function, thus depending only upon the rate of growth and the workers' saving propensity, the remaining ratios and the rate of profit depend on s_W / g_n and on the form of production function.

Indeed, they insist upon the generality of their formal analysis. Though most of the above seems to rest on marginal productivity notions, they argue that no direct use was actually made of marginal productivity relations. For instance, there would be no necessity to identify the profit-rate relations $r = f'(k)$ with $df(k)/dk$. Indeed, all we need is that r should be a determinate function of K/L and the that function need not be the above derivative. In their own words,

> *even if there are no smooth substitutability properties posited for [the given] production function or even if Chamberlain imperfect competition intervenes in factor or commodity markets, our analysis can still be applied. If Kalecki, or Boulding, or Hahn, or Kaldor, or Schneider, or Walter Reuther, or Thünen come forward with some alternative theory of distribution, provided only that the profit rate is a declining function of the ratio of capital to intensive – call it* $r = \phi(K/L)$ *– both the Pasinetti formalism and our various duals and generalizations of them remain valid* (Samuelson and Modigliani, 1966a, p. 287).

In his reply, Pasinetti's (1966b) main contention is that their analysis has a serious drawback, namely, it was written with the aim of defending a specific theory – the theory of marginal productivity of capital. As such, therefore, it has compelled SM to fit Pasinetti's new result within the rigid constraints of a preconceived framework, his (1966b) purpose then being to point out the quite restrictive consequences of such an approach.

He claims this new result is that – whether or not we believe in the existence of something called marginal productivity of capital – the long-run equilibrium rate of profit – with the proviso that $s_W < I/Y$ – turns out to be determined according to the relation $P/K = (1/s_c)/(I/K)$, which is fully independent of marginal productivity assumptions. For Pasinetti, SM, though admitting that the marginal productivity theory of capital has become unnecessary to explain the long-run profit rate (an admission which he infers from SM's remark quoted in the previous paragraph), actually pursue a second line of defense. That is, he sees their contribution as an attempt to show that, though the "paradox" (*i. e.* the relation above) is true, it is not incompatible with the marginal productivity theory. The reason is that, if one does believe in marginal productivity and if one is willing to make the required assumptions, one can always claim that, in the long run, the marginal productivity of capital will become equal to the (independently determined) profit rate. That is, the rate of profit, in the long run, determines what the marginal productivity of capital is going to be.

For Pasinetti, these arguments are

> *unconvincing, since marginal productivity is a* concept*, which was invented in order to explain the rate of profit. Professors Samuelson and Modigliani now seem to turn the problem round and aim at using the rate of profit in order to explain and justify the concept of marginal productivity.*
> *The whole procedure seems to me artificial and unnecessary. It looks like building a superfluous and complicated scaffolding around a construction which stands on its own.*
> (Pasinetti, 1966b, p. 303-304, original emphasis).

He begins by recalling the precise limits within which the Cambridge equation is valid, namely, $s_W < I/Y = k(I/K)$. As regards the above-mentioned contention by SM that such restriction is ill-defined because I/K refers to equilibrium situations and because (in their analysis, though not in Pasinetti's, according to the latter's view) it is not a given of the problem but a characteristic of the solution, he replies as follows:

> *It is exactly for this reason that I find 'ill-defined' their own formulation of the same condition, namely, $S_W < S_c\, P/Y$, where P/Y is indeed referred to equilibrium situations and is not a given of the problem but a characteristic of the solution (this time in* their own *analysis, as well as in mine).*
> (Pasinetti, 1966b, p. 304, f. 1, original emphasis).

In any case, he recognizes that when $s_W = I/Y$, there is a correspondent growth path on which the proportion of the total capital stock owned by the workers tends to unity. When one looks at these problems from the point of view of marginal productivity theory, it is added, it is almost inevitable that one should become attracted (as SM do) by this particular case, simply because, by having only one category of savers, it becomes analogous to the case treated in neoclassical economic models. Let us recall that SM quoted two specific reasonable figures for P/Y and S_c (respectively, 0.25 and 0.20), thus concluding that the condition for the validity of the Pasinetti's theorem would cease to be satisfied for s_W any higher than a modest 5 percent. Pasinetti, in turn, by taking figures supposedly closer to those observed, shows that that the critical level for s_W would in fact be of the order of 12 to 16 percent.

But let us leave aside the details of their alternative calculations and focus upon the theoretical side of the controversy. Following Pasinetti, let us suppose that the proviso above were not to be satisfied. Yet, to suppose so does not yet mean that s_W and I/Y should be exactly equal; s_W might well be more than I/Y. Actually, SM take a further step, for they add an assumption borrowed from marginal productivity theory, namely, that the capital-output ratio is a smooth monotonic declining function of the profit rate within the considered range. If the usual limitations are imposed that such parameters should be nonnegative, this range goes in fact from zero to infinity, which means infinite possibilities of substitution between capital and intensive in response to opposite changes in the profit rate

Thanks to this assumption, he adds, cases given by $s_W > I/Y$ are excluded, the reason being that I/Y, when it is not less than S_W, always becomes equal to s_W. In the analysis set forth by SM, therefore, either $s_W < I/Y$ or $s_W = I/Y$, for there can never be $S_W < I/Y$. But then he correctly raises the question of whether this assumption may be justified. Given that the capital-output ratio is a macroeconomic magnitude which is composed of all the prices and all the physical quantities of commodities, Pasinetti argues that there appears to be no theoretical justification at all for assuming that, in general, that ratio should be a monotonic decreasing function of the rate of profit (let alone that it should be a smooth function, and that it should go from zero to near infinity). Pasinetti claims that this assumption is crucial to that part of SM's analysis which relies on I/Y becoming equal to s_W *i. e.* to their Dual Theorem so that the latter, though appearing formally symmetrical to Pasinetti's, is actually substantially not symmetrical to it. While it depends crucially on that assumption, thus breaking down any time that it does not hold, Pasinetti's theorem is valid independently of it.

As regards SM's contention that his theorems have no peculiar relevance to a Kaldorian alternative theory of distribution or to some version of a Cambridge theory of distribution, Pasinetti, though agreeing with them that his formulation of the Cambridge equation is one of great generality, correctly adds that one should not infer from this that it is irrelevant for such theories. According to Pasinetti, his analysis is rather a necessary ingredient of all of them; in particular, it is relevant to Kaldor's theory of income distribution more than to any other.

In her pertinent comment on SM, Robinson (1966) focuses upon their analysis of the limits within which a Pasinetti golden age is actually possible. She begins by recalling that, in any period, net profit is equal to net investment plus the excess of consumption out of profits over savings out of wages, a relation implied by the following identities:

$$Y \equiv P + W \equiv I + C \equiv I + (1 + s_p)P + (1 - s_W)W$$

Pasinetti divides s_p into two parts, s_c which applies to profits accruing to capitalists, rK_c, and s_W which applies to profits on the part of capital owned by workers, rK_W, r being uniform on all capital. In a Pasinetti equilibrium, with the share of capital owned by each class equal to its share in net saving, the extra expenditure out of profits accruing to workers (due to the excess of s_c over s_W) offsets savings out of wages. Profits are then equal to investment plus expenditure by capitalists and the rate of profitis equal to g/s_c.

Robinson argues that when there is a choice of a known technique (which according to her at M.I.T. means differences in the quantity of putty-capital per man employed) and it is postulated that investment is embodied in profit-maximizing form, we can draw up a pseudo-production function (Solow's name for it) showing output per man employed, real wage, and value of capital per man, corresponding to each profit rate. Then in a Pasinetti golden age, with $g = g_n$ and $r = g_n/s_c$, we can find the appropriate value of K/Y and so determine the profit share, rK/Y. In her view, SM (following Meade) are able to strike a blow for K/Y by showing that it enters into the determination of the limits within which a Pasinetti golden age is actually possible.

Let us recall that Pasinetti stated the condition as $s_W < I/Y$. Robinson argues that this is perfectly correct, but it does not bring out the effect of the share of wages in income on the share of workers' saving in investment so that the condition is most perspicuously stated as $s_W Y < s_c rK$. If $s_W Y > s_c rK$, equilibrium with the profit rate constant through time requires the whole capital to be owned by workers for, if the proportion of capital owned by workers is changing as total capital increases, the rate of profit will not be constant. Then two classes of savers do not exist and the system reduces itself to a Harrod equilibrium with $Y = I/s$. For a Harrod-equilibrium golden age, however, K/Y must be equal to s/g_n. In order to show that a Harrod equilibrium is possible for all values of s and g_n, SM assume a well-behaved pseudo-production, with K/Y – a smooth, continuous decreasing function of the rate of profit. For Robinson (1966, p. 290-291, f. 1), though SM admit that there is no logical reason why the pseudo-production function should be of this form, they just assume that it is so, an assumption that leads her to conclude:

> *After putting the rabbit into the hat in full view of the audience it does not seem necessary to make so much fuss about drawing it out again.* (Robinson, 1966, p. 308).

But it is not sufficient, she adds, to postulate that K/Y can assume any required value, so that a question which arises regards to whether there is some mechanism to cause it to be equal to s/g_n. In her view, this poses no problem for SM, for they think of savings as consisting in accumulating putty. When s/g_n is greater than K/Y, putty per man employed is rising, the rate of profits falling and K/Y rising, and contrariwise when K/Y is greater than s/g_n. In other words, in a Harrod-type golden age equilibrium, putty or not, profit is equal to $I+(1-s)P-sW$, but in order to know the rate of profits, P/K, we have to look at the pseudo-production function to find K. Actually, K/Y is equal to s/g_n, simply because, if it were not, this would not be a Harrodian golden age. Thus, given s, g_n and a well-behaved pseudoproduction there is one rate of profit at which a Harrod golden age is not impossible; if the function is badly behaved there may be several or none. With Pasinetti's assumptions, on the other hand, there can be a golden age at any rate of profit with any pseudoproduction function provided that $s_W Y < s_c rK$; when $s_W = 0$, a golden age is possible at any profit rate. Indeed, such alternative (to Pasinetti's above mentioned) demonstration of a clear asymmetry between Pasinetti's theorem and SM's anti-Pasinetti theorem is one of the most interesting feature of Robinson's intervention in the debate.

Kaldor's (1966) comment on SM was as well intended to point out the unrealistic nature of most of the assumptions behind their theoretical results. In their analysis, he maintains, there is no room for phenomena such as increasing returns, learning by doing, oligopolistic competition, uncertainty, obsolescence and other such troublesome ones that mar the world as we know it. As regards SM's contention that any macroeconomic theory, which makes use of the notion of differences in savings propensities between profits and wages, requires an identifiable class of hereditary barons – that is, a class of capitalists with permanent membership – distinguished by a high savings propensity and as well as of a permanent class of workers, distinguished by a low savings propensity, Kaldor replies that he always regarded the high savings propensity out of profits as something which attaches to the nature of business income and not to the wealth (or other peculiarities) of the individuals who own property.

In his view, it is the enterprise, not the particular body of individuals owning it at any one time, which finds it necessary, in a dynamic world of increasing returns, to plough back a proportion of the profits earned as a kind of "prior charge" on earnings in order to ensure the survival of the enterprise in the long run. According to him, this is sufficient to refute SM's contention that, provided the savings propensity of workers is high enough, the capitalists (distinguished by their high savings propensity) will be gradually eliminated so that, in a golden-age equilibrium, only one savings propensity is left. Let us recall that in order to show that, SM consider a situation in which the basic Pasinetti inequality, namely, that the share of investment in total income is higher than the share of savings in wages (or in total personal income) does not hold as regards the equilibrium level of investment. Ironically enough, it is concluded, the end of it all is not a violent revolution, à la Marx, but the cosy world of Harrod, Domar, and Solow, where there is only a single savings propensity applicable to the economy.

For Kaldor, the simple response to all this is that, if the Kaldor-Pasinetti inequality is not satisfied, no Keynesian macrotheory of distribution could survive for an instant, let alone in a golden-age equilibrium. In other words, if the equilibrium level of investment were less than the workers' savings, it is impossible to contemplate that investment should play the active role and savings the passive role; for, if we assumed that investment decisions were autonomous, either the full employment assumption would break down or profits would have to be negative and, in either case, it is clearly inconceivable that profits should be determined by the need to generate sufficient savings to finance investment. Kaldor then concludes that

> *[it is easy to refute Pasinetti by postulating conditions in which the Pasinetti model could not possibly work, and where therefore something else must take its place (...) Samuelson and Modigliani assume, as a matter of course, that it must be Walras. In disproving Pasinetti they conjure up a Walrasian world in all its purity – a world in which all savings get invested somehow, without disturbing full employment]* (Kaldor, 1966, p. 312).

As regards to SM's recourse to some reasonable figures for P/Y and s_c (0.25 and 0.20, respectively) in order to show that the condition for the validity of the Pasinetti's theorem would cease to be satisfied for s_W higher than 5 percent, Kaldor argues that in their demonstration they make several slips, so that their conclusion does not follow.

He is equally skeptical about SM's claim that their results do not depend on marginal productivity notions. Let us recall that the claim, which all their results require, is the postulate that the profit rate should be a single-valued function, ϕ, of the capital-intensive ratio, with $\phi' < 0$. For Kaldor, what is problematic about their claim is that no reason whatever is adduced to show why such assumption is any less restrictive than the whole bag of tricks specified in the neoclassical formulation of their model, in which they postulate a (constant return to scale) production function, $y = f'(k)$, with $f'(k) > 0$ and $f'(k) < 0$ as well as neoclassical smoothness and substitutability and perfect markets, under which conditions competition will enforce equality of factor prices to factor marginal productivities. According to Kaldor, the assumption of a functional relation between the rate of profits and the capital-labor ratio is implied in the assumption underlying that bag of tricks, it being purely arbitrary without them. Nor is any attempt made to support the validity (or plausibility) of such an assumption empirically. For Kaldor, K/L, unlike K/Y, showed the widest of variations between the different countries – it was perhaps twenty times as high in the U.S. as in India – whilst the rate of profits was often to be found to be higher in countries with a relatively high K/L than with a low one.

Finally, he develops a Neo-Pasinetti theorem which is argued to hold in any steady growth rate and does not postulate a class of hereditary capitalists with a special high-saving propensity. Let us recall that the Pasinetti theorem shows that, under certain conditions, the rate of profits, in a true long-run, golden-age equilibrium, does not depend on the workers' savings rate, because the additional consumption out of the workers' property income will offset their savings out of wage income. Kaldor claims that the difficulty with this proposition (apart from the fact that it would occur in the "very long run") is that it assumes that workers spend the same fraction of their income, irrespective of whether it accrues to them as property income or wages. However, in a world where enterprises are organized as corporations, and property income takes the form of dividends, this would imply overspending their dividend income by the exact fraction required to make their consumption equal to $(1 - s_W)P_W$, irrespective of the division of profits between corporate retention and dividends. Besides, once we allow spending in excess of divided income, there is no reason to confine such spending to workers, for capitalists also spend some part of their capital gains, or even their capital, in the absence of such gains.

Thus, at any time there must be capitalists (or shareholders) who overspend their current (dividend) income (and the same must be true, of course, of retired workers who consume their accumulated savings over the years of retirement) just as there are active workers who save a certain fraction of their income for retirement. Just as net savings out of income sets up a demand for securities, net disavings out of income (= net consumption out of capital or capital gains) sets up a supply of securities. There is also a net supply of new securities issued by the corporate sector. Since, in the security market, prices will tend to a level at which the total (nonspeculative) supply and demand for securities are equal, there must be some mechanism to ensure that the spending out of capital (or capital gains) just balances the savings out of income less new securities issued by corporations.

As for the details of the model, he divides the community into wage and salary earners, W, who save some fraction of their income during their working life and consume it in retirement. As long as the population is rising and income per head is rising, the savings of the working population must exceed the disavings of the retired population by an amount which can be expressed as some fraction, s_W, of current wage-and-salary income. He also assumes that the shareholders' net consumption out of capital (*i. e.* their consumption in excess of their dividend income) is some fraction, c, of their capital gains, G. Finally, he supposes that corporations, having decided on retaining a fraction, s_c, of their profits, decide in addition to issue new securities equal to some fraction, i, of their current investment expenditure, gK. Thus, equilibrium in the security market requires that $s_W W = cG + igK$, which means that at least one of these items must be responsive to changes in the market value of securities. Such an item is cG, since G is nothing else than the change in the market value of securities, and it varies not only with the rise in dividends and earnings per share, but also with the valuation ratio, v, *i. e.* the relation of the market value of shares to the capital employed by corporations (or the "book value" of assets).

After several algebraic manipulations, Kaldor (1966) derives solutions for the profit rate and the valuation ratio whose interpretation is as follows. Given the savings coefficients and the capital-gains-consumption coefficient, there will be a certain valuation ratio which will secure just enough savings by the personal sector to take up the new securities issues by corporations. Thus, the net savings of the personal sector (available for investment by the business sector) will depend not only on the savings propensity of individuals, but also on the policies of the corporations towards new issues. In the absence of new issues the level of securities will be established at the point at which the purchases of securities by savers will be just balanced by the sale of securities of the disavers, making the net savings of the personal sector equal to zero.

The issue of new securities by corporations will depress security prices (*i. e.* the valuation ratio) just enough to reduce the sale of securities by the disavers sufficiently to induce the net savings required to take up the new issues. If i were negative and corporations were net purchasers of securities from the personal sector, the valuation ratio would be driven up to the point at which net personal savings would be negative to the extent necessary to match the sale of securities to the corporate sector. It should be mentioned that Kaldor's analysis assumes that savings out of dividends are zero; cG is seen as the net excess of shareholders' consumption over dividend income.

In a golden-age equilibrium (given a constant g, and a constant K/Y), v will be constant, with a value that can be greater or smaller than 1, depending upon the values of s_c, S_W, c and i. In fact, all that one can eventually assert is that, given the Pasinetti inequality, $gK > s_W Y$, $v < 1$ when $c = (1 - s_W)$, $i = 0$; with $i > 0$ this will be true a fortiori. Thus, the profit rate in a golden-age equilibrium, $P/K = g(1-i)/s_c$, will be independent of the personal savings propensities, s_W and c. It is in this way that Kaldor's Neo-Pasinetti theorem is similar to the original Pasinetti theorem, though it is reached by a different route. Besides, it will hold in any steady-state growth, and not only in a long-run golden age; it does not postulate a class of hereditary capitalists with a special high-saving propensity. In the special case given by $i=0$, it reduces to Pasinetti´s version of the Cambridge equation given by $P/K = g/S_c$.

Let us now turn to SM's (1966b) reply to Pasinetti's (1966b) and Robinson's (1966) comments on their neoclassical reformulation of Pasinetti's hypothesis. It is worthy of mention that they do not reply to Kaldor (1966), the alleged reason being that Kaldor's paper reached them too late. In any case, SM begin by recalling that the major motivation for undertaking the original paper stemmed from the perception that the Pasinetti golden-age equilibrium, instead of being the general one, had to be recognized as but one of two golden-age equilibria, being matched by a Dual or Anti-Pasinetti one. As is the usual case for duality relations, SM adds, there is complete symmetry between the Primal and Dual equilibria, in the sense that neither is more general than the other. Unlike Pasinetti, who strongly insisted that his golden-age equilibrium is a more general one, being relevant quite independently of marginal productivity assumptions and well-behaved functional relationships between profit rates and capital-output ratios, they claim that the existence of the Dual golden age has nothing to do with those assumptions.

In this context, the main purpose of their reply is to demonstrate that the symmetry of generality between the Dual and Primal regimes does definitely hold for any multiple-blueprint technology of the kind that Robinson and MIT economists think useful to analyze, thus establishing once and for all that it does not depend on any simple diminishing returns assumptions of the neoclassical type. In providing this constructive demonstration, SM add, they are also able to isolate that special technological case – which is believed to be realistic – would provide considerable justification for concentrating on the Pasinetti regime to the exclusion of the Dual on the ground that the latter is a knife-edge solution.

For simplicity, they assume a single consumption good and a finite number, large or small, of different blueprint pages, each corresponding to a different activity or capital process. As is well known, each profit rate(?) excludes many pages of the blueprints as not being competitively viable, leaving one or more sets of activities that can be viable at the given golden-age profit rate. They stress that nothing well-behaved is assumed about technology other than that the factor-price frontier relating real wage and profit rate(?) must be downward-sloping, so that this frontier may have changing curvatures and along it there may be reswitching effects of the Cohen-Sraffa type, shifts toward lower capital-output ratios as profit rate falls, and any kind of Wicksell effects. No singular equal-factor-intensity assumptions are made that might validate any surrogate capital concepts.

Next, they assume a natural rate of labor growth of, n, that is positive, for simplicity of exposition ignoring Harrod-neutral technical change. The question arising then regards what golden-age configurations can prevail for this natural rate of growth at each profit rate, r. They argue that there will emerge, for fixed n and each r, an admissible configuration of processes and price ratios: hence, and this is what matters for their argument, there will be for given n an admissible set of capital values and ratios of aggregate capital value to value of output, which could be plotted against profit rates. For SM, it is convenient to work with the reciprocal of the aggregate capital-output ratio, which is a percentage per annum, but is now quite divorced from any physical capital of jelly or surrogate type, being merely the ratio of value of total market stocks of capital to value of total output.

After several algebraic and graphical manipulations, SM are in position to draw a figure – with s_c and s_W plotted on the horizontal and vertical axis, respectively – which shows the division of the region of savings coefficients into the Primal or Pasinetti equilibrium region and the Dual equilibrium region. Their procedure is the following. On a first figure, they plot the above-mentioned average product of capital against the profit rates for a general blueprint technology. On another figure, they introduce a saving behavior of the Pasinetti type, with s_c and s_W plotted on the horizontal and vertical axis, respectively. Thus, from any point on this latter figure, with its specified (s_c, s_W)values, and with n given, they go back to the former figure and find the corresponding golden-age equilibrium point or points. It is their contention that this can be done for a general blueprint technology; and when it is done, they find that there are two symmetrical regions, corresponding to the Primal and Dual equilibria.

As for Robinson's comment on their contribution, their reply is not extensive. They interpret her main contention – that they did put the rabbit into the hat in full view of the audience before drawing it out again – as meaning that their logical theorems do follow correctly from their axiomatic conditions; in their view, this is a fact for self-congratulation not apology! According to them, Robinson's further implication, namely, that their logical proofs of stability and existence are so transparently obvious as to involve a trivial waste of time, reveals more what she considers tiresome than an objective finding. Besides, they argue that in her (incomplete) summary of their analysis, there is naught for them to quarrel with or to give comfort to Pasinetti's critique. What is useful in her comment being the remainder that the one-sector leets model does have special properties that must not be extrapolated to more general models. In their reply, however, they claim to have returned good for good, showing graphically what happens to existence problems in a general blueprint technology.[2]

4_ Closing remarks

Kaldor's (1955-1956) delivered a consistent solution to Harrod´s long-run problem of having the warranted growth rate, which is given by the ratio of the average propensity to save to the capital-output ratio, to equal the natural growth rate, which is given by the growth rate of labor supply plus the rate of technological change. Though the growth rate is rather assumed to be at Harrod´s natural rate, functional income distribution is determined by the requirement that

[2] Several authors, though not discussed here, have subsequently extended and modified the Kaldor-Pasinetti approach by considering different rates of return for capitalists and workers, more general saving functions, internal financing of investment by firms, varying capacity utilization and employment, financial assets, government fiscal policy, and open economy issues – see Baranzini (1991) for an early account, and Panico and Salvadori (1993) for a re-publication of some of the relevant papers; see also Commendatore *et al.* (2003) for a recent account. Indeed, this large literature has shown that the Pasinetti paradox obtains under a much wider range of conditions – though not under all – than those considered initially by Pasinetti himself.

savings is equal to autonomous investment, with Harrod´s long-run problem being solved through changes in the average propensity to save brought about by changes in income distribution.

Pasinetti (1962) correctly argued that Kaldor had neglected to take into account the fact that wage earners who saved would have two sources of income, namely from wages and from returns to capital wealth. Nonetheless, Pasinetti developed a growth model which shows the irrelevance of workers' propensity to save while uncovering the very strategic importance of the decisions to save of capitalists, a result which has been taken to demonstrate the inability of workers to directly influence income distribution in the long run. While the duality theorem derived by Samuelson and Modigliani (1966) did seem to restrict the generality of Pasinetti's analysis, the neo-Pasinetti theorem that Kaldor delivered in rebuttal did come to the rescue of the post- Keynesian approach to distribution based on (some variant of) the Cambridge equation.

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